Exploiting Structure in Compressed Sensing Using Side Constraints – From Analysis To System Design (EXPRESS II)

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Research Objective

EXPRESS I: Exploit structure in measurement system $\mathbf{A}$, sparsity $\mathbf{x}(t)$ and representation $\mathbf{x}$.
EXPRESS II: From Analysis to System Design:

- **Signal model:**

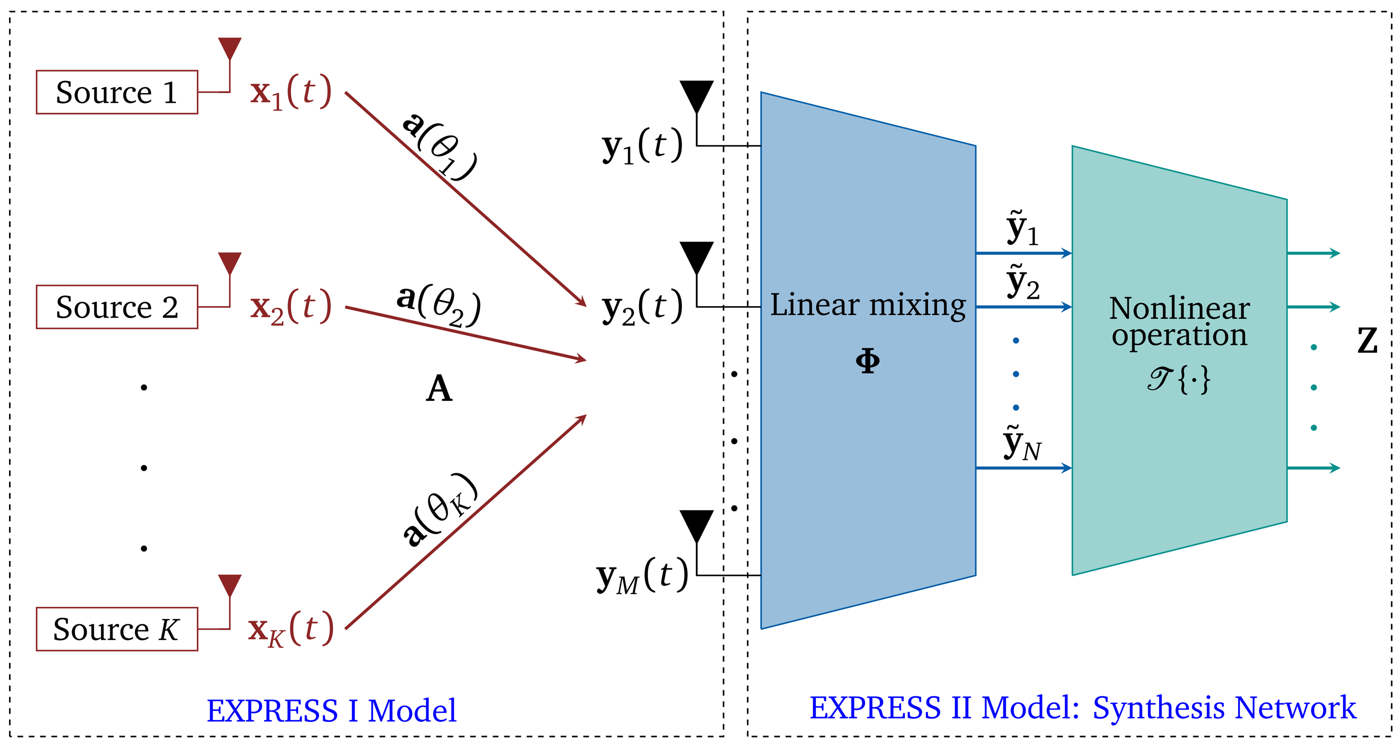
$$\mathbf{Z} = \mathcal{T}\left\{\mathbf{\Phi}^{(0)}\mathbf{A}(\boldsymbol{\theta}^{(0)})\mathbf{X}^{(0)}\right\} + \mathbf{N}$$

- **Problem of interest:**

$$\min_{\mathbf{X},\mathbf{\Phi}} \ \frac{1}{2}\|\mathbf{Z} - \mathcal{T}\{\mathbf{\Phi}\mathbf{A}(\boldsymbol{\theta})\mathbf{X}\}\|_F^2 + \lambda\|\mathbf{X}\|_{p,q} \ : \ \textit{side constraints} \qquad \text{(P0)}$$

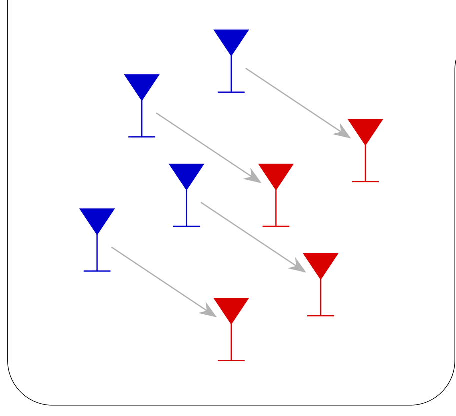
- **Examples for $\mathcal{T}$ and $\mathbf{\Phi}$:**
  - quantization (K-Bit, One-Bit, Sigmoid, ...),
  - constant modulus ($|x_i| = c_i \in \mathbb{R}$),
  - nonlinear, mixing or compression network.

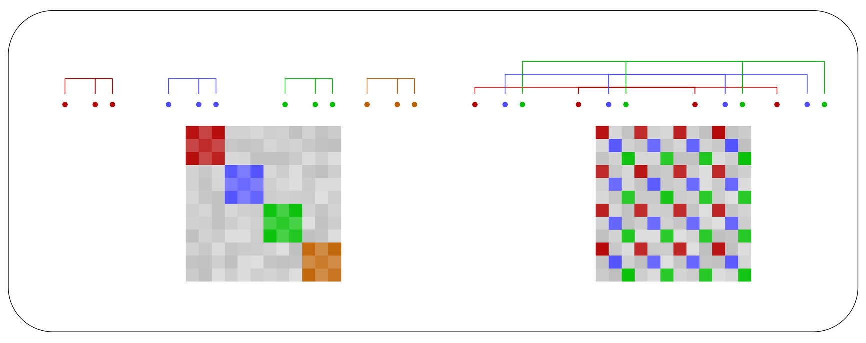
Hybrid Linear-Nonlinear Measurement System

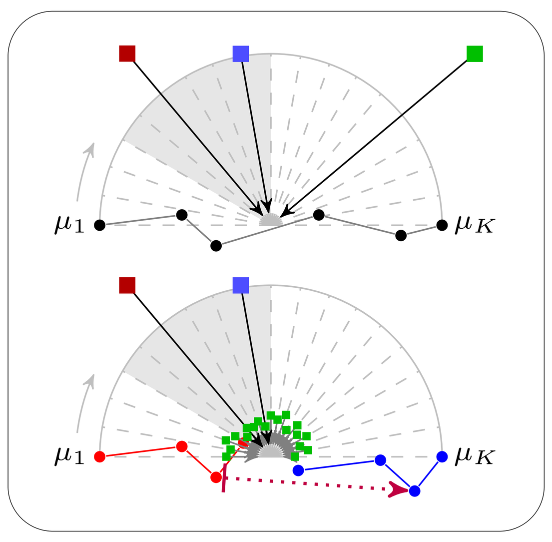


Exploiting Different Types of Structure

- Shift invariance: $\mathbf{J}_1\mathbf{a}(\theta) = \mathbf{J}_2\mathbf{a}(\theta)z(\theta)$

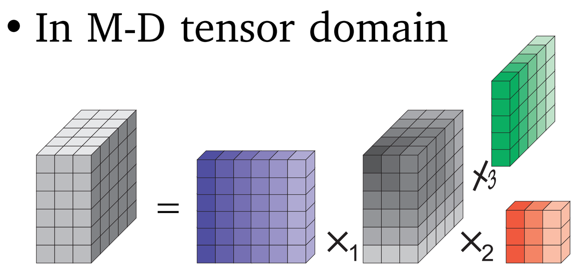






WP 1: structure of measurement system $\mathbf{A}$

- In M-D tensor domain



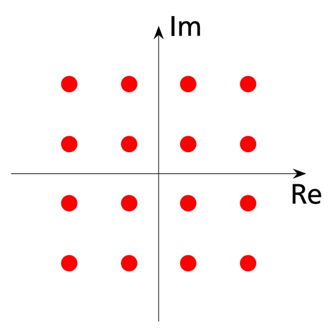
- Constellation signals and integrality property:

$$\min_{\mathbf{x}} \{\|\mathbf{x}\|_0 \ : \ \mathbf{A}\mathbf{x} = \mathbf{y}, \ \mathbf{l} \le \mathrm{Re}(\mathbf{x}), \ \mathrm{Im}(\mathbf{x}) \le \mathbf{u}, \ \mathrm{Re}(\mathbf{x}), \ \mathrm{Im}(\mathbf{x}) \in \mathbb{Z}^N\},$$

$$\min_{\mathbf{x}} \{\|\mathbf{x}\|_0 \ : \ \mathbf{A}\mathbf{x} = \mathbf{y}, \ \mathbf{x} \in \{-1,0,1\}^N\},$$

$$\min_{\mathbf{x}} \{\|\mathbf{x}\|_0 \ : \ \mathbf{A}\mathbf{x} = \mathbf{y}, \ \mathbf{x} \in \{0,1\}^N\}$$

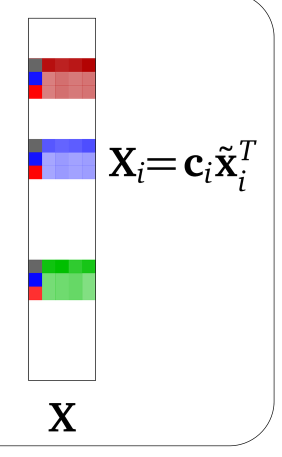
$$\min_{\mathbf{x}} \{\|\mathbf{x}\|_0 \ : \ \mathbf{A}\mathbf{x} = \mathbf{y}, \ \mathbf{l} \le \mathbf{x} \le \mathbf{u}, \ \mathbf{x} \in \mathbb{Z}^N\}$$



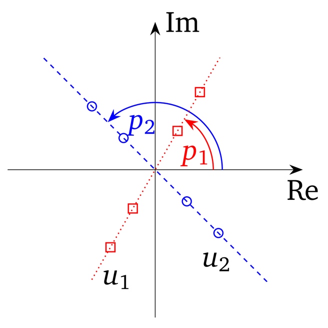
WP 0: structure of mixing $\mathbf{\Phi}$ and nonlinearity $\mathcal{T}$

- Partly calibrated array:

$$\min_{\mathbf{X},\mathbf{b}} \ \sum_{i=1}^{N} b_i \quad \text{s.t.} \quad \mathbf{A}\mathbf{X} = \mathbf{Y}, \quad \mathrm{rank}\,\mathbf{X}_i = b_i, \quad \mathbf{b} \in \{0,1\}^N$$



- Non-circular sources

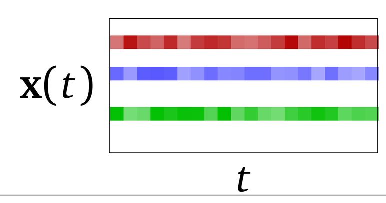


- Constant modulus SDP-relaxation:

$$\min_{\mathbf{x},\mathbf{X},\mathbf{w}} \sum_{j=1}^{N} w_j$$
$$\text{s.t.} \ \mathbf{A} \bullet \mathbf{X} - 2\,\mathbf{z}^T\mathbf{A}\mathbf{x} + \mathbf{z}^T\mathbf{z} \le \delta,$$
$$\Gamma := \begin{pmatrix} \mathbf{X} & \mathbf{x} \\ \mathbf{x}^T & 1 \end{pmatrix} \succeq 0,$$
$$\mathrm{rank}(\Gamma) = 1, \ \mathrm{diag}(\mathbf{X}) = \mathbf{w}, \ \mathbf{w} \in \{0,1\}^n,$$

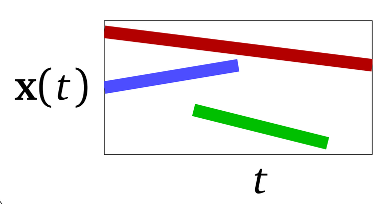
WP 2: structure of representation $\mathbf{x}$

- Group sparsity in one-dimensional arrays $x_i(1) = \ldots = x_i(D) = 0$



WP 3: structure of sparsity $\mathbf{x}(t)$

- Tracking applications



Highlights from EXPRESS I

Exploiting Structure for Algorithms

- SPARROW: Grid-less sparse recovery procedure for MMP with multiple shift-invariant arrays and fully-augmentable arrays [1, 2]
- COBRAS: lift 2-D sparse reconstruction problems (e.g. 2-D DoA estimation) into 1-D block- and rank-sparse reconstruction problem [3, 2]
- BOSE: fast and accurate estimation scheme to exploit the block-sparse tensor structure [4]
- STELA: pseudoconvex approx. algorithm framework for nonsmooth optimization [5, 6]
- Cramér-Rao bound on the estimation accuracy of a Bernoulli-distributed block-sparse core tensor in the context of CS with Gaussian measurement matrices [4]
- Specialized branch-and-cut algorithm for complementarity & cardinality constraints [7, 8]

Recoverability

- Constellation and integral signals [9]
- Gridless sparse recovery in multiple measurement problem [10, 11]
- Gridless sparse recovery based on atomic norm minimization for strictly NC sources [12]
- Cramér-Rao bound for strictly NC signals as a benchmark for the maximum achievable gain from sparse NC signal recovery [13]

Achievements and Activities

- 8 journal papers and 11 conference papers published, 2 preprints submitted, 2 dissertations successfully completed: [7, 2], Tutorials at IEEE ICASSP 2015 and EUSIPCO 2017, Special session on "Exploiting structure in compressed sensing" at IEEE CAMSAP 2017

List of Publications from EXPRESS I

[1] C. Steffens, M. Pesavento, and M. E. Pfetsch, "A compact formulation for the $\ell_{2,1}$ mixed-norm minimization problem," *Transactions on Signal Processing*, vol. 66, no. 6, pp. 1483–1497, 2018.

[2] C. Steffens, "Compact formulations for sparse reconstruction in fully and partly calibrated arrays," Ph.D. dissertation, TU Darmstadt, 2017.

[3] C. Steffens and M. Pesavento, "Block- and rank-sparse recovery for direction finding in partly calibrated arrays," *IEEE Transactions on Signal Processing*, pp. 384–399, 2018.

[4] R. Boyer and M. Haardt, "Noisy compressive sampling based on block-sparse tensors: Performance limits and beamforming techniques," *IEEE Transactions on Signal Processing*, no. 23, pp. 6075–6088, 2016.

[5] Y. Yang and M. Pesavento, "A unified successive pseudoconvex approximation framework," *IEEE Transactions on Signal Processing*, vol. 65, no. 13, pp. 3313–3328, 2017.

[6] C. Steffens, Y. Yang, and M. Pesavento, "Multidimensional sparse recovery for MIMO channel parameter estimation," in *Proc. 24th European Signal Processing Conference (EUSIPCO)*, Budapest, Hungary, 2016, pp. 66–70.

[7] T. Fischer, "Branch-and-cut for complementarity and cardinality constrained linear programs," Ph.D. dissertation, TU Darmstadt, 2017.

[8] T. Fischer and M. E. Pfetsch, "Branch-and-cut for linear programs with overlapping SOS1 constraints," *Mathematical Programming Computation*, 2017, to appear.

[9] J.-H. Lange, M. E. Pfetsch, B. M. Seib, and A. M. Tillmann, "Sparse recovery with integrality constraints," arXiv, Tech. Rep. 1608.08678, 2016, http://arxiv.org/abs/1608.08678.

[10] J. Steinwandt, C. Steffens, M. Pesavento, and M. Haardt, "Sparsity-aware direction finding for strictly non-circular sources based on rank minimization," in *Proc. IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM)*, Rio de Janeiro, Brazil, 2016.

[11] J. Steinwandt, F. Roemer, and M. Haardt, "Sparsity-based direction-of-arrival estimation for strictly non-circular sources," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Shanghai, China, 2016.

[12] J. Steinwandt, F. Roemer, C. Steffens, M. Haardt, and M. Pesavento, "Gridless superresolution direction finding for strictly non-circular sources based on atomic norm minimization," in *Proc. Asilomar Conference Signals, Systems, and Computers*, Pacific Grove, CA, 2016.

[13] J. Steinwandt, F. Roemer, M. Haardt, and G. Del Galdo, "Deterministic Cramér-Rao bound for strictly non-circular sources and analytical analysis of the achievable gains," *IEEE Transactions on Signal Processing*, vol. 64, no. 17, pp. 4417–4431, 2016.

See also www.projekt-express.tu-darmstadt.de.

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